



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$= -\frac{(a+ex_1)[a^2y_1y_1+b^2x_1x_2-a^2b^2]}{a^2(a+ex_1)[x_1y_2-x_2y_1-ae(y_1-y_2)]} = -\frac{a^2y_1y_2+b^2x_1x_2-a^2b^2}{a^2[x_1y_2-x_2y_1-ae(y_1-y_2)]}.$$

It is seen that this expression is symmetrical with reference to x_1 and x_2 , y_1 and y_2 with the exception of the sign, but considering that by finding $\tan PFB$ the slope of BF comes first, it is at once seen that $\tan PFB$ is the same as $\tan PFA$. The difficulty of this method lies in the complicated algebraic work, which is avoided by using polar coördinates.

Solution of 255 by Prof. William Hoover was received after the solution in last issue had gone to press. Also a solution of 256 was received from a contributor who failed to sign his name.

NOTE. Professor Matz sent in a solution of 254 in which he points out that the line $x-4a=0$ is both tangent and normal to the curve. But the solution is not general. Who can give a general solution?

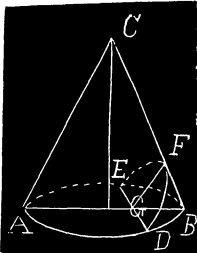
CALCULUS.

195. Proposed by CHRISTIAN HORNING, Heidelberg University, Tiffin, O.

Given a right cone of altitude h and radius r , to locate the plane parallel to its side which bisects the cone.

Solution by A. H. HOLMES, Brunswick, Maine, and J. SCHEFFER, Hagerstown, Md.

Let, in the right cone CAB , DEF represent a parabolic section. Put $BG = x$, $GE = y$, $FG = z$. The area of $DEF = \frac{4}{3}yz$; and consequently the volume of



$$BDEF = \frac{4}{3} \cdot \frac{h}{\sqrt{(r^2 + h^2)}} \int_0^x yz dx,$$

and since $y^2 = 2rx - x^2$, and $z = \frac{x}{2r} \sqrt{(r^2 + h^2)}$, we have for the volume, the integral

$$\frac{2}{3} \frac{h}{r} \int_0^x x dx \sqrt{(2rx - x^2)} = \frac{2}{3} \frac{h}{r} \left[\frac{1}{2} r^3 \cos^{-1} \frac{r-x}{r} - \frac{3r^2 + rx - 2x^2}{6} \sqrt{(2rx - x^2)} \right].$$

To determine x for the condition that this volume is to be half the cone, we have the transcendental equation

$$2r^3 \cos^{-1} \frac{r-x}{r} - \frac{2}{3} (3r^2 + rx - 2x^2) \sqrt{(2rx - x^2)} = r^3.$$

An approximate value of x is $x = 1.3r$.

Also solved by R. D. Carmichael.

196. Proposed by F. P. MATZ, Sc. D., Ph. D., Reading, Pa.

The shortest tangent intercepted by the axes of the ellipse to which the tangent is drawn, equals the sum of the semi-axes of the ellipse.

I. Solution by the PROPOSER.

Tangent $= y\sqrt{1+(dx/dy)^2} + x\sqrt{1+(dy/dx)^2}$, in which $y = (b/a) \times \sqrt{a^2 - x^2}$ and $dy/dx = -bx/a\sqrt{a^2 - x^2}$.

$$\begin{aligned}\therefore U &= \frac{1}{a} \left[\frac{\sqrt{a^2 - x^2}}{x} + \frac{x}{\sqrt{a^2 - x^2}} \right] \sqrt{a^4 - (a^2 - b^2)x^2} \\ &= a \sqrt{\left(\frac{a^4 - (a^2 - b^2)x^2}{x^2(a^2 - x^2)} \right)} = a \text{ minimum.}\end{aligned}$$

$\therefore x = a^3/(a+b)$; and, consequently, the length of the required tangent becomes as stated in the problem.

II. Solution by G. W. GREENWOOD, M.A., Professor of Mathematics, McKendree College, Lebanon, Ill., and J. SCHEFFER, Hagerstown, Md.

Denote the length of the tangent by l , and its equation by $y = mx + \sqrt{a^2m^2 + b^2}$.

$$\therefore l^2 = \left(1 + \frac{1}{m^2}\right)(a^2m^2 + b^2).$$

$$l^2m^2 = a^2m^4 + b^2 + a^2m^2 + b^2m^2 = (am^2 - b)^2 + m^2(a+b)^2.$$

$$\therefore l^2 = (a+b)^2 + \left(\frac{am^2 - b}{m}\right)^2.$$

Hence the minimum value of l is $a+b$.

Also solved by M. E. Graber, and W. L. Tryon.

197. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

$$\int_0^\infty \frac{\sin mx \cos nx}{x} dx.$$

Solution by G. W. GREENWOOD, M. A., Lebanon, Ill.; M. E. GRABER, Tiffin, Ohio, and the PROPOSER.

The required integral may be written

$$\frac{1}{2} \int_0^\infty \left[\frac{\sin(m+n)x}{x} + \frac{\sin(m-n)x}{x} \right] dx,$$

and it therefore reduces to problem No. 186, [January, 1905, page 22]. If $m+n$ and $m-n$ are both positive, the result is $\frac{1}{2}\pi$. If both negative, $-\frac{1}{2}\pi$. If of opposite sign, 0.

Also solved by S. A. Corey.

189. Proposed by M. E. GRABER, A. M., Heidelberg University, Tiffin, O.

Show that $e \int_1^\infty, e^2 \int_2^\infty, \dots, e^n \int_n^\infty, \dots$ are integers divisible by $(p+1)!$, when the expression under the integral is $x^p [(x-1) \dots (x-n)]^{p+1} e^{-x} dx$.